TRIANGULAR AND TRAPEZOIDAL FUZZY STATE ESTIMATION WITH UNCERTAINTY ON MEASUREMENTS

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Abstract. *In this paper, a fuzzy state estimation(FSE) model is employed, which is based on constrained linear programming (LP) optimization, for modelling uncertainty in power system state estimation. The estimation process is based on uncertainty measurements. The uncertain measurements are expressed as fuzzy numbers with a triangular and trapezoidal membership function that has middle and spread value reflected on the estimated states. The proposed fuzzy model is formulated as a linear optimization problem, where the objective is to minimize the sum of the spread of the states, subject to double inequality constraints on each measurement. Linear programming technique is employed to obtain the middle and the asymmetric spread for every state variable. The estimated middle corresponds to the value of the estimated state, while the asymmetric spreads represent the tightest uncertainty interval around that estimated states. The proposed formulation has been applied to various test systems such as, 6-bus, IEEE 14-bus and IEEE30-bus.*

Keywords

Fuzzy membership function, linear programming, quadratic programming, state estimation, uncertainty.

1. Introduction

An important aspect of power system operation is the availability of an accurate picture of the system-state. A state estimator can be used to filter the available information creating an accurate and complete picture of the system conditions, while a supervisory control and data acquisition (SCADA) system is capable of providing operators with measured information with less accuracy. The redundancy available in the measurement systems are traditionally used to reduce the effect of measurement errors using state estimation. The objectives of state estimation methods are to reduce the variance of the estimates and improve their overall accuracy, detection of gross errors, invalid topological information and model parameter errors.

If the errors in a measurement follow a known probability distribution, a set of feasible estimates can be modeled by a probability distribution function. It is unfortunately difficult to characterize statistics of observation errors in practice. In such circumstances, it is desirable to provide not just a single ‘optimal’ estimate of each state variable but also an uncertainty range within which we can be assured that the ‘true’ state variable must lie. The idea of an uncertainty range is recognizable in engineering practice, where the accuracy of a particular measurement is often described in percent e.g. plus or minus 2 %, rather than by quantifying the standard deviation or variance.

Introduced the concepts of uncertainty in the general context of engineering analysis, estimation and optimization [1]. These concepts have been extended and developed and are applied in several areas, e.g. in water distribution networks.

With an intention to increase the robustness of the estimation introduced bounds on the measurements [2]. The approach has been developed, who introduced the term set, bounded state estimation (SBSE) [3]. The concepts from robust control theory and allowed for uncertainty in both the parameters and the measurements has applied [4]. Using a linear fractional transformation the uncertainty is isolated and the problem is formulated as a convex semi definite programming problem. The semi definite programming problem is solved using a linear matrix inequalities approach. For modeling uncertainty in power system state estimation proposed a fuzzy linear state estimation model based on Tanaka’s fuzzy linear regression model [8]. The uncertainty is modeled via deterministic upper and lower bounds on measurement errors, which take into account known meter accuracies [6].

In conventional state estimation techniques, the accurate knowledge of error statistics of transducers and metering equipments is a prerequisite. However, as such information may not be precisely known it can lead to less accurate estimates. The overall quality of the estimation can be improved by providing additional information using estimated bounds together with the point estimates. Knowledge of the limiting values or bounds that apply to measured quantities facilitates a problem formulation that enables the computation of bounds on state estimates. Thus, the goal of this paper is to model the uncertainties associated with the measured quantities in a way that defines an interval (range) with respect to their nominal values. The range is governed by the tolerance of the measuring instrument (a quantification of accuracy usually provided by the manufacturer). By utilizing appropriate mathematical programming techniques, the confidence interval (or bounds) of the state variables can be computed.

2. Fuzzy Logic

Fuzzy logic is an artificial intelligence tool that has been used in the past decade for many control applications. Fuzzy logic emerged from fuzzy set theory founded [7], [8] by challenging basic assumptions of these theories: sharp boundaries in classical set theory, classical logic that each proposition must be either true or false, and additivity in classical measure theory, particularly probability theory.

Unlike classical logic systems, fuzzy logic aims at modeling the imprecise modes of reasoning, which is the human ability to make a rational decision when information is uncertain and imprecise.

Fuzzy logic starts with the concept of a fuzzy set. A fuzzy set is a set without a crisp, clearly defined boundary. It can contain elements with only a partial degree of membership. Membership criteria are not precisely defined for most classes of objects normally encountered in the real world. A fuzzy set F is characterized by a membership function, μ, that takes values in the interval [0, 1], such that the nearer the value of μ (x) to unity, the higher the membership grade of x in F.

3. Problem Formulation

The nonlinear equations relating the measurements and the state vector may be expressed as:

$$Z = \mathrm{h}(Y) + \mathrm{e}\,. \tag{1}$$

For a given set of measurements Z, the exact value of Y cannot be determined. What we require, instead, is to find the optimal estimate of Y denoted by $\hat{Y}$.

Equation (1) can be linearized around some operating point Y_0 to yield the following relationship:

$$\Delta Z = H(Y_0).\Delta Y + \boldsymbol{e}\,. \tag{2}$$

$$\Delta Z = Z - h(Y_0)\,. \tag{3}$$

$$\Delta Y = \hat{Y} - Y_0\,. \tag{4}$$

$$H(Y_0) = \frac{\partial h(Y_0)}{\partial Y}\,. \tag{5}$$

The approach adopted in WLS state estimation is to minimize a weighted sum of some function of the residuals [9], [10], [11], [12], [13], [14], [15]. If we denote the absolute value of the k^{th} measurement residual by R_k, then the goal of WLS estimation is to minimize the vector of residuals *R:*

$$R = [R_1\ R_2\ R_3\ \ldots\ R_m]\,. \tag{6}$$

$$R_m = \left|\Delta Z_k - H(Y_0).\Delta Y\right|. \tag{7}$$

4. Uncertainty Interval State Estimation via Linear Programming (UILP)

Uncertainty can be filed with the solution of a series of optimization problems formulated is determined appropriate. Any angle or voltage bus, together with its associated uncertainty, can by a triangular or trapezoidal membership functions representing. In the triangular membership function Y_1 and Y_3 are lower and upper bounds for the central value (Y_2). These constraints define the tolerances on the measurements (i.e. the range of values within which the true value of the measurements quantity should lay). Minimizing a particular state variable of interest, subject to the entire measurements inequality constraints, provides the lower bound on that state variable.

Maximizing the state variable, again subject to the entire measurements inequalities, provides the upper bound for that state. In mathematical form:

$$\begin{gathered}\min_{Y}\ \mathrm{Y}_i \\ \text{subject to } Z^l \le h(Y) \le Z^u\end{gathered}\,. \tag{8}$$

$$Z^l = Z - \delta^-\,. \tag{9}$$

$$Z^u = Z + \delta^+\,. \tag{10}$$

Equation (8) defines a nonlinear constrained optimization problem, which can be solved directly by a suitable nonlinear programming algorithm such as sequential quadratic programming [17]. However, it is known that power system models are amenable to solution using the WLS approach. Consequently, an alternative approach is to linearise Eq. (8) about a suitable point $\hat{Y}$ (which in this case can be provided by the WLS) and then a series of linear programming are solved to obtain updates dY_i to the uncertainty bounds on the state variables. For example, the incremental change to the lower bound for the ith state can be computed by solving the following LP problem:

$$\begin{gathered}\min_{\Delta Y} \ dY_i \\ \text{subject to } \ \Delta Z^l \leq J\Delta Y \leq \Delta Z^u\end{gathered} . \tag{11}$$

Similarly, the incremental change to the upper bound on the ith state can be found by solving the LP problem

$$\begin{gathered}\max_{\Delta Y} \ dY_i \\ \text{subject to } \ \Delta Z^l \leq J\Delta X \leq \Delta Z^u\end{gathered} . \tag{12}$$

Where J is the Jacobian of $h(Y)$ evaluated at $\hat{Y}$ and ΔZ^l and ΔZ^u are vectors of the incremental changes to measurements lower and upper bounds, respectively, computed in the following form:

$$\Delta Z^l = Z^l - h(\hat{Y}) . \tag{13}$$

$$\Delta Z^u = Z^u - h(\hat{Y}) . \tag{14}$$

Therefore by performing 2n linear programming solutions, all the elements of the vectors $dY+$ and $dY-$ can be calculated. Once $dY+$ and $dY-$ are known, the bounds on $\hat{Y}$ are simply found as:

$$Y^+ = \hat{Y} + dY^+ , \tag{15}$$

$$Y^- = \hat{Y} + dY^- , \tag{16}$$

where $\hat{Y}$ is the point obtained by WLS.

The computational burden of the process arises from the need to perform two LP solutions for every uncertainty interval sought. Nevertheless, with the measurement redundancy level available in power systems, the computational time is reasonable using modern hardware and software. For large networks it is possible that the dual LP formulation could be applied to reduce the execution time [18], [19].

5. Fuzzy State Estimation

In this paper, the basic procedure for obtaining the membership function of fuzzy voltages and angles, in the triangular membership measurements (Fig. 1), a WLS is solved for the central values (Y_2), and then the variation around them (Y_1 and Y_3) are calculated using fuzzy arithmetic and linear programming. Similarly, for the trapezoidal membership measurements (Fig. 2), WLS is solved for the inner breakpoints (Y_2 and Y_3) and then the outer breakpoints (Y_1 and Y_4) are calculated using fuzzy arithmetic and linear programming.

This approach was applied for fuzzy power flow [16].

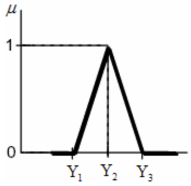

Fig. 1: Triangular fuzzy distribution.

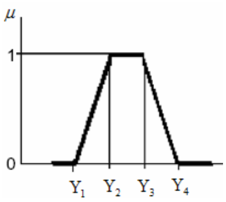

Fig. 2: Trapezoidal fuzzy distribution.

It is noticeable in sections 7 and 8, for all estimated values, triangular and trapezoidal functions like Fig. 1 and Fig. 2 are obtained.

6. Implementation of Proposed Method and Result Analysis

In this section some results obtained using the proposed algorithms typical test system 6 - bus as shown in Fig. 3 [20], 30 - data bus test network. All state variables will be calculated to show the concepts of the present approach is shown. The LP problems have been solved by the function linprog incorporated in the MATLAB™ optimization toolbox.

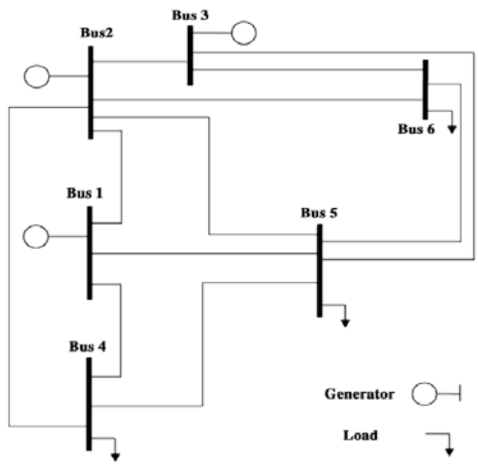

Fig. 3: Online diagram of six-bus system.

7. Triangular Fuzzy State Estimation Analysis with UILP

In order to demonstrate the ability of the proposed algorithm, the state estimation solutions for 6-bus (shown in Fig. 3) and modified IEEE 30-bus test systems are presented under uncertainty of measurements. In order to save space, 6-bus system results are given in detail, whereas 30-bus system results are given only with trapezoidal distribution uncertainties.

Cases I and II: Tables 1and 2 show the triangular fuzzy state estimation for the 6-bus and IEEE-14 network, respectively. The measurements uncertainty is represented as a uniform distribution over the interval [-5 % 5 %] of the nominal value of the measurements. A WLS was used to compute the central point (Y_2) states.

In the tests presented here and in further tests the Newton–Raphson process was found to perform reliably, with convergence occurring within three or four iterations.

The outer breakpoints in triangular fuzzy distribution (Y_1 and Y_3) of the state variables were found using Eq. (8)–(14).

8. Trapezoidal Fuzzy State Estimation with UILP

For applying the trapezoidal fuzzy distribution (Fig. 2), first the inner breakpoints (Y_2 and Y_3) were obtained for angle and magnitude of voltages using fuzzy arithmetic and WLS. Then, outer breakpoints (Y_1 and Y_4) were calculated. (Y_1) calculated in Eq. (11) and Eq. (16), and Y_4 in Eq. (12) and Eq. (15).

Cases III and IV: Tables 3 and 4 illustrate the trapezoidal fuzzy distribution for magnitude and angle of voltages that fuzzy measurements include trapezoidal and trapezoidal-triangular membership function, respectively.

Cases V and VI: Results for 14 and 30-bus test system with uncertainties in the measurements are shown in the Tables 5 and 6, respectively.

The results indicate that for a particular breakpoint of a variable of interest, the FSE finds out a specific mismatch vector from input variable vector and evaluated function vector in case of uncertainty in measurements, in the universe of discourse defined by the range of uncertainty in these variables.

8. Advantages and Practicalities THE Fuzzy State Estimation with UILP

The availability of the triangular or trapezoidal membership function on state estimates can have practical advantages for the power system operator. For Critical quantities, such as a power flow which is close to its thermal, stability or contractual limit, the operator can gain confidence that the true value is not exceeding the constraint provided that the state estimate and both bounds are all within the limit. The uncertainty range on the estimate also gives a useful indication of the quality of the metering configuration for the relevant part of the power system. For example, where a voltage level often has a wide estimated uncertainty range, this would suggest that the metering in that area is insufficient. This type of additional information could be very useful during the installation or upgrading of an online state estimator. In addition with the introduction of measurement variation in the formulation, a more realistic and accurate uncertainty range is attainable now about the different system quantities.

9. Trapezoidal Fuzzy State Estimation Losses and Active Line Flow

The fuzzy membership functions losses and active line flow obtaining with voltage (with consideration the relations of the voltage and power) and fuzzy arithmetic. Tables 7 and 8 depict fuzzy distribution (breakpoint values) of real power losses, while Tab. 9 display distribution of the real line flows on a few sample lines due to fuzziness in the measurements for 6 bus. The variations in the measurements are in the interval [-5 % 5 %] nominal measurements.

Tab.1: Results for case I.

Bus No.	V (p.u.)			θ (°)		
	V_1	V_2	V_3	θ_1	θ_2	θ_3
1	1,0500	1,0500	1,0500	0	0	0
2	1,0302	1,0497	1,0692	-3,6535	-3,5891	-3,5246
3	1,0552	1,0742	1,0931	-4,5935	-4,5003	-4,4072
4	0,9680	0,9888	1,0096	-4,3083	-4,1854	-4,0624
5	0,9675	0,9883	1,0092	-5,3642	-5,2034	-5,0426
6	0,9890	1,0108	1,0325	-6,3648	-6,1782	-5,9915

Tab.2: Results for case II.

Bus No.	V (p.u.)			θ (°)		
	V_1	V_2	V_3	θ_1	θ_2	θ_3
1	1,0600	1,0600	1,0600	0,0000	0,0000	0,0000
2	1,0370	1,0450	1,0560	-5,2293	-5,0038	-4,3549
3	1,0060	1,0074	1,0084	-13,1232	-12,7371	-11,5703
4	1,0142	1,0173	1,0228	-10,4027	-10,2679	-8,9939
5	1,0214	1,0214	1,0266	-8,9916	-8,8356	-7,6640
6	1,0693	1,0754	1,0789	-14,1907	-14,1901	-12,1236
7	1,0543	1,0584	1,0651	-13,4726	-13,4000	-11,0023
8	1,0812	1,0821	1,0847	-13,9072	-13,5457	-11,1273
9	1,0520	1,0546	1,0637	-15,0083	-15,0074	-12,7096
10	1,0448	1,0498	1,0606	-15,2139	-15,1361	-12,4138
11	1,0468	1,0548	1,0638	-15,0462	-14,8574	-12,4069
12	1,0528	1,0643	1,0721	-15,2529	-14,9322	-12,7149
13	1,0491	1,0569	1,0634	-15,1145	-14,9969	-12,8288
14	1,0367	1,0464	1,0602	-16,1773	-15,8186	-13,1964

Tab.3: Results for case III.

Bus No.	Crisp solution		Fuzzy state estimation solution							
	|V| (p.u.)	θ (°)	|V| (p.u.)				θ (°)			
			V_1	V_2	V_3	V_4	θ_1	θ_2	θ_3	θ_4
1	1,0500	0	0,9983	1,0319	1,0703	1,0893	0	0	0	0
2	1,0497	-3,5891	0,9859	1,0272	1,0739	1,0969	-5,2058	-3,5828	-3,5090	-2,5968
3	1,0742	-4,5003	1,0072	1,0430	1,1055	1,1340	-6,4462	-4,4918	-4,4742	-3,7867
4	0,9888	-4,1854	0,9328	0,9708	1,0112	1,0375	-5,1548	-4,1688	-4,1389	-3,2675
5	0,9883	-5,2034	0,9397	0,9616	1,0146	1,0570	-5,4611	-5,1974	-5,1730	-4,1354
6	1,0108	-6,1782	0,9559	0,9796	1,0422	1,0828	-6,5440	-6,2000	-6,1213	-5,1792

Tab.4: Results for case IV.

Bus No.	Crisp solution		Fuzzy state estimation solution							
	|V| (p.u.)	θ (°)	|V| (p.u.)				θ (°)			
			V_1	V_2	V_3	V_4	θ_1	θ_2	θ_3	θ_4
1	1,0500	0	0,9996	1,0316	1,0702	1,0889	0,0000	0,0000	0,0000	0,0000
2	1,0497	-3,5891	1,0079	1,0492	1,0501	1,0731	-5,2853	-3,5898	-3,5884	-2,6038
3	1,0742	-4,5003	1,0372	1,0730	1,0754	1,1039	-6,4822	-4,5103	-4,4904	-3,7853
4	0,9888	-4,1854	0,9519	0,9877	0,9899	1,0140	-5,1567	-4,1707	-3,5898	-3,3287
5	0,9883	-5,2034	0,9675	0,9873	0,9894	1,0297	-5,4448	-5,2256	-5,1811	-4,1881
6	1,0108	-6,1782	0,9876	1,0103	1,0113	1,0509	-6,4974	-6,2029	-6,1535	-4,8609

Tab.5: Results for case V.

Bus No.	V (p.u.)				θ (°)			
	V_1	V_2	V_3	V_4	θ_1	θ_2	θ_3	θ_4
1	1,0600	1,0600	1,0600	1,0600	0,0000	0,0000	0,0000	0,0000
2	1,0312	1,0450	1,0965	1,1023	-5,4542	-5,1612	-4,9414	-4,2850
3	1,0062	1,0100	1,0795	1,0981	-13,1273	-12,7434	-12,6257	-11,4456
4	1,0103	1,0158	1,0889	1,0945	-10,4064	-10,3578	-10,2300	-9,1191
5	1,0100	1,0179	1,0929	1,0982	-9,1395	-8,9438	-8,7872	-7,9560
6	1,0505	1,0700	1,1584	1,1590	-14,2057	-14,2041	-14,1463	-12,0554
7	1,0515	1,0552	1,1450	1,1518	-13,4753	-13,4055	-13,3383	-10,9219
8	1,0834	1,0900	1,1698	1,1601	-13,9112	-13,5527	-13,5184	-11,0795
9	1,0462	1,0487	1,1391	1,1483	-15,0360	-15,0285	-14,7706	-12,4521
10	1,0379	1,0417	1,1342	1,1451	-15,2383	-15,1525	-14,8071	-12,0646
11	1,0383	1,0464	1,1392	1,1482	-15,0545	-14,8659	-14,7242	-12,2516
12	1,0309	1,0424	1,1483	1,1562	-15,2578	-14,9367	-14,9072	-12,6629
13	1,0330	1,0408	1,1410	1,1475	-15,2612	-15,0054	-14,9484	-12,7536
14	1,0116	1,0213	1,1300	1,1439	-16,1799	-15,8270	-15,7404	-13,0928

9. Conclusion

A new analysis of uncertainty in fuzzy estate estimation is presented in this paper. The uncertainty is assumed to be present in the measurements that have fuzzy membership functions (triangular or trapezoidal). The uncertainty in the output (angle and magnitude of voltages) was obtained from the fuzzy arithmetic and linear programming method. The advantage of this method is that one can assume different fuzzy membership functions for measurements of fuzzy state estimation whether triangular, trapezoidal or trapezoidal-triangular.

When applied and tested on different systems standard proposed estimator can be very effective as a tool for estimating the confidence interval unknowns and their uncertainty and imprecision due to be considered. Convergence and evaluation based on time, estimates indicated that this support can be a valuable tool in estimating the power line system used state.

Tab.6: Results for case VI.

Bus No.	Crisp solution		Fuzzy state estimation solution							
	|V| (p.u.)	θ (°)	|V| (p.u.)				θ (°)			
			V_1	V_2	V_3	V_4	θ_1	θ_2	θ_3	θ_4
1	1,0200	0,0000	0,9762	0,9984	1,0492	1,0672	0	0	0	0
2	1,0000	-0,4154	0,9529	0,9826	1,0124	1,0191	-0,5082	-0,4154	-0,4154	-0,3225
3	0,9826	-1,5218	0,9318	0,9631	0,9944	1,0018	-1,6888	-1,5218	-1,5218	-1,3548
4	0,9796	-1,7949	0,9292	0,9604	0,9916	0,9991	-1,9719	-1,7949	-1,7949	-1,6180
5	0,9823	-1,8637	0,9349	0,9635	0,9922	1,0025	-2,1154	-1,8637	-1,8637	-1,6121
6	0,9728	-2,2671	0,9225	0,9536	0,9848	0,9922	-2,4974	-2,2671	-2,2671	-2,0368
7	0,9671	-2,6519	0,9176	0,9479	0,9782	0,9866	-2,9243	-2,6519	-2,6519	-2,3795
8	0,9602	-2,7259	0,9095	0,9411	0,9727	0,9795	-2,9836	-2,7259	-2,7259	-2,4681
9	0,9804	-2,9967	0,9296	0,9608	0,9921	1,0000	-3,2122	-2,9967	-2,9967	-2,7812
10	0,9845	-3,3748	0,9346	0,9649	0,9953	1,0042	-3,5252	-3,3748	-3,3748	-3,2243
11	0,9804	-2,9974	0,9267	0,9608	0,9949	1,0000	-3,2435	-2,9974	-2,9974	-2,7514
12	0,9853	-1,5370	0,9337	0,9657	0,9977	1,0052	-1,5443	-1,5370	-1,5370	-1,5298
13	0,9996	1,4763	0,9477	0,9797	1,0117	1,0196	1,2895	1,4763	1,4763	1,6631
14	0,9765	-2,3083	0,9249	0,9570	0,9890	0,9960	-2,4265	-2,3083	-2,3083	-2,1901
15	0,9802	-2,3119	0,9289	0,9606	0,9922	0,9998	-2,3129	-2,3119	-2,3119	-2,3109
16	0,9770	-2,6446	0,9258	0,9575	0,9892	0,9966	-2,7534	-2,6446	-2,6446	-2,5358
17	0,9771	-3,3922	0,9270	0,9576	0,9881	0,9967	-3,5392	-3,3922	-3,3922	-3,2452
18	0,9687	-3,4786	0,9179	0,9494	0,9808	0,9881	-3,5084	-3,4786	-3,4786	-3,4487
19	0,9654	-3,9582	0,9144	0,9461	0,9778	0,9847	-4,0182	-3,9582	-3,9582	-3,8982
20	0,9690	-3,8711	0,9178	0,9496	0,9814	0,9884	-3,9187	-3,8711	-3,8711	-3,8234
21	0,9932	-3,4887	0,9444	0,9737	1,0030	1,0131	-3,6891	-3,4887	-3,4887	-3,2882
22	0,9998	-3,3928	0,9510	0,9802	1,0094	1,0198	-3,5838	-3,3928	-3,3928	-3,2018
23	1,0000	-1,5893	0,9482	0,9800	1,0119	1,0199	-1,6506	-1,5893	-1,5893	-1,5279
24	0,9891	-2,6315	0,9380	0,9694	1,0009	1,0089	-2,7378	-2,6315	-2,6315	-2,5252
25	0,9905	-1,6900	0,9368	0,9706	1,0044	1,0103	-1,9732	-1,6900	-1,6900	-1,4068
26	0,9719	-2,1397	0,9168	0,9524	0,9879	0,9913	-2,3970	-2,1397	-2,1397	-1,8824
27	1,0001	-0,8282	0,9462	0,9802	1,0142	1,0201	-1,1444	-0,8282	-0,8282	-0,5119
28	0,9747	-2,2657	0,9237	0,9550	0,9863	0,9941	-2,5207	-2,2657	-2,2657	-2,0108
29	0,9799	-2,1285	0,9214	0,9603	0,9992	0,9995	-2,5592	-2,1285	-2,1285	-1,6978
30	0,9678	-3,0417	0,9072	0,9484	0,9872	0,9896	-3,4689	-3,0417	-3,0417	-2,6145

Tab.7: Results for case VII.

Crisp P_{loss} (p.u.)	Fuzzy distribution (p.u.)			
	P_{FL1}	P_{FL2}	P_{FL3}	P_{FL4}
0,0777	0,0669	0,0750	0,0850	0,0900

Tab.8: Results for case VIII.

Crisp P_{loss} (p.u.)	Fuzzy distribution (p.u.)			
	P_{FL1}	P_{FL2}	P_{FL3}	P_{FL4}
0,0279	0,0084	0,0266	0,0271	0,0514

Tab.9: Fuzzy distribution of real state estimation on simple lines of 6-bus system.

Line No.	Crisp state estimation (p.u.)	Fuzzy distribution (p.u.)			
		P_{FL1}	P_{FL2}	P_{FL3}	P_{FL4}
3(1-5)	0,3502	0,3270	0,3483	0,3521	0,3740
5(2-4)	0,3922	0,3171	0,3511	0,4445	0,4791

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